

Isospin symmetry breaking and baryon-isospin correlations from Polyakov–Nambu–Jona-Lasinio model

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We present a study of the 1+1 flavor system of strongly interacting matter in terms of the Polyakov–Nambu–Jona-Lasinio model. We find that though the small isospin symmetry breaking brought in through unequal light quark masses is too small to affect the thermodynamics of the system in general, it may have significant effect in baryon-isospin correlations and have a measurable impact in heavy-ion collision experiments.

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Signatures of phases of matter with deconfined color charges is under critical investigation for last few decades, both theoretically and experimentally. Quantum Chromodynamics (QCD) is the formulation for first principle studies of strongly interacting matter. Along with the local color symmetry the quark sector has a few global symmetries. In the chiral limit for two light flavors u and d , we have global vector and axial vector symmetry $SU_V(2) \otimes SU_A(2)$. For non-zero quark masses, the axial symmetry $SU_A(2)$ is explicitly broken, while for non-zero quark mass difference vector (isospin) symmetry $SU_V(2)$ is explicitly broken. At low energies the isospin symmetry breaking (ISB) has relevance in many aspects of hadronic observables [1]. Apart from the quark mass difference, ISB effects may be brought in by electromagnetic contributions as well. Low energy $\pi - K$ scattering has been studied considering the inclusion of electromagnetic correction into the effective Lagrangian [2]. ISB of valence and sea quark distributions in protons and neutrons has been studied in the chiral quark model [3, 4]. ISB may also have significant effect in the context of existence of CP violating phase [5]. Some Lattice QCD investigation of the effect of unequal quark masses was done in Ref.[6] and recently the effect of ISB on different hadronic observables were studied in Ref.[7, 8]. Within the framework of chiral perturbation theory the isospin breaking effect in quark condensates has been studied considering $m_u \neq m_d$ and electromagnetic corrections as well, where the authors have given an analysis of scalar susceptibilities [9, 10].

In the context of high energy heavy ion collisions where strongly interacting matter is supposed to exist in a state of thermal and chemical equilibrium, the ISB effects have not been explored much. Fluctuations and correlations of conserved charges are important and sensitive probes

for heavy ion physics. Most of the theoretical studies in this respect are in isospin symmetric limit (see *e.g.* [11–27]). Here we present our first case study of ISB effect on fluctuations and correlations of strongly interacting matter within the framework of the Polyakov loop enhanced Nambu–Jona–Lasinio (PNJL) model. We discuss the possible experimental manifestations of the ISB effects based on quite general considerations in the limit of small current quark masses.

Formalism: In the last few years PNJL model has appeared in several forms and context to study the various aspects of phases of strongly interacting matter (see *e.g.* [18, 19]). Here we use the form of the 2 flavor PNJL model with the thermodynamic potential studied by us in [20, 21], but with a mass matrix,

$$\hat{m} \equiv m_1 \mathbb{1}_{2 \times 2} - m_2 \tau_3 \\ = \begin{pmatrix} m_1 - m_2 & 0 \\ 0 & m_1 + m_2 \end{pmatrix} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}.$$

where, $\mathbb{1}_{2 \times 2}$ is the identity matrix in flavor space and τ_3 is the third Pauli matrix. While a non-zero m_1 breaks the chiral $SU_A(2)$ symmetry explicitly a non-zero m_2 does the same for the isospin $SU_V(2)$ symmetry. For the case at hand m_2 is also the difference of constituent masses of u and d quarks.

We restrict ourselves to the mean field approximation and investigate the behavior of different charge susceptibilities. Once the pressure P is obtained, the n^{th} order diagonal and off-diagonal susceptibilities are given by, $\chi_n^X = \frac{\partial^n (P/T^4)}{\partial (\mu_X/T)^n}$ and $\chi_{ij}^{XY} = \frac{\partial^{i+j} (P/T^4)}{\partial (\mu_X/T)^i \partial (\mu_Y/T)^j}$ with $i + j = n$. These susceptibilities are related to the fluctuations and correlations of arbitrary conserved charges X and Y with corresponding chemical potentials μ_X and

μ_Y . Given a two flavor system the global charge conservation is expected for baryon B , isospin I and electric Q charges. Since in the isospin symmetric limit the $B - I$ correlations vanish exactly, we shall study this correlation with explicit isospin breaking.

Results and Discussions: Here we consider the average quark mass $m_1 = (m_u + m_d)/2$ fixed at 0.0055 GeV and study the effect of ISB with three representative values of $m_2 = (m_d - m_u)/2$. The parameter set in the NJL sector has been determined separately for the different values of m_2 . The differences in the parameter values were found to be practically insignificant. The bulk thermodynamic properties of the system expressed through pressure, energy density, specific heat, speed of sound etc. did not show significant dependence on m_2 . Even the diagonal susceptibilities were almost identical to those at the isospin symmetric limit. However, interesting differences were observed for the off-diagonal susceptibilities in the $B - I$ sector. We first discuss the $\mu_B = 0$ case. We shall consider $\mu_I = 0$ in this work. It should be noted that even for $\mu_I = 0$, a non-zero m_2 would generate some non-zero isospin number.

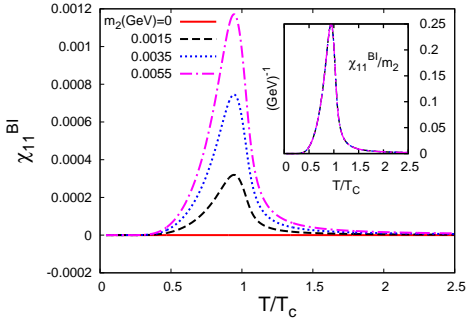


FIG. 1. Second order off-diagonal susceptibility in $B - I$ sector at $\mu_B = 0$.

In Fig.1 the second order off-diagonal susceptibility χ_{11}^{BI} , is plotted against T/T_c for different values of m_2 . Here T_c is the crossover temperature obtained from the inflection point of the scalar order parameters - the mean values of chiral condensate and Polyakov Loop [19–21]. As expected we find $\chi_{11}^{BI} = 0$ for $m_2 = 0$. For non-zero m_2 the non-monotonic behavior observed here can be understood as follows. At low temperatures the fermionic excitations are suppressed due to their large constituent masses as well as the suppression due to the Polyakov loop. On the other hand at high temperatures the mass difference m_2 become insignificant compared to the temperature scale. Therefore only at some intermediate temperatures one can expect a non-zero χ_{11}^{BI} . The peak value appears very close to T_c .

The sensitivity of χ_{11}^{BI} on m_2 is clearly visible. An exciting feature observed here is the almost linear scaling of χ_{11}^{BI} with m_2 . This is shown in the inset of Fig.1.

At the fourth order we have the off-diagonal suscepti-

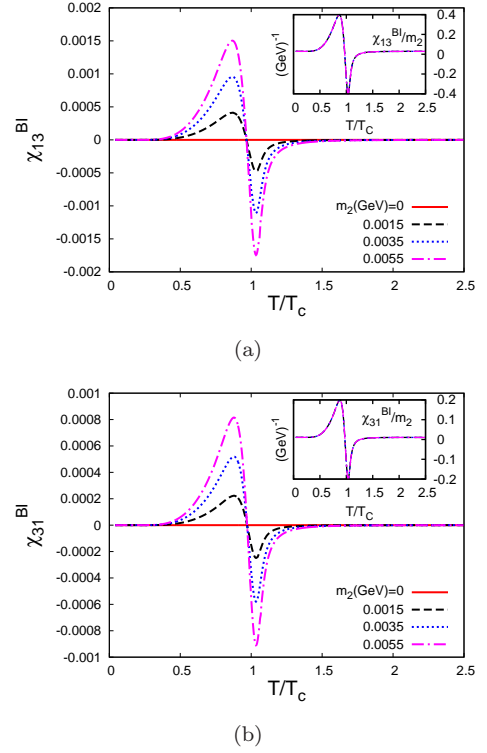


FIG. 2. Fourth order off-diagonal susceptibilities.

bilities, χ_{13}^{BI} , χ_{31}^{BI} and χ_{22}^{BI} . For $\mu_B = 0$ the T dependence for the first two quantities and their linear scaling with m_2 is shown in Fig.2. For χ_{22}^{BI} we found no such scaling behavior and the m_2 dependence was insignificant.

Qualitatively one can understand the behavior of χ_{13}^{BI} and χ_{31}^{BI} by noticing that these are correlators between the fluctuations χ_2^I of isospin and χ_2^B baryon number respectively, with the $B - I$ correlator χ_{11}^{BI} . In our earlier studies [19–21] we found that the both the fluctuations increase monotonically with temperature. Here we found in Fig.1 above that with increase in T , χ_{11}^{BI} first increases and then decreases with a turning point close to T_c . Thus χ_{11}^{BI} is correlated with χ_2^I and χ_2^B below T_c and is anti-correlated above. Therefore one can expect that the correlations given by χ_{13}^{BI} and χ_{31}^{BI} are positive and negative respectively below and above T_c .

To understand the presence of m_2 scaling for some correlators and absence in others we first note that the different $B - I$ correlators may be expressed in terms of those in the flavor space. The corresponding relation between the chemical potentials are $\mu_u = \frac{1}{3}\mu_B + \frac{1}{2}\mu_I$ and $\mu_d = \frac{1}{3}\mu_B - \frac{1}{2}\mu_I$. This implies,

$$\chi_{11}^{BI} = \frac{1}{6}(\chi_2^u - \chi_2^d). \quad (1)$$

The flavor diagonal susceptibilities can be expanded in a Taylor series of the quark masses around $m_u = m_d = 0$.

$$\chi_2^f(m_u, m_d) = \sum_{n=0}^{\infty} \sum_{i=0}^n a_{i,j}^f m_u^i m_d^j \quad (2)$$

where, $a_{i,j}^f = \frac{1}{i!j!} \left[\frac{\partial^n \chi_2^f}{\partial m_u^i \partial m_d^j} \right]_{m_u=m_d=0}$ are the Taylor coefficients, with $i+j=n$ and $f \in u, d$. Here $a_{0,0}^u$ and $a_{0,0}^d$ are respectively u and d flavor susceptibilities in the chiral limit; hence they are equal. Moreover, response of χ_2^u to a change in m_u (m_d) and that of χ_2^d to a change in m_d (m_u) are identical in the chiral limit. Thus we have $a_{i,j}^u = a_{j,i}^d, \forall i, j$. Therefore we get,

$$\begin{aligned} & \chi_2^u(n^{th} \text{ order}) - \chi_2^d(n^{th} \text{ order}) \\ &= \sum_{i=0}^n \alpha_i m_u^i m_d^i (m_d^{n-2i} - m_u^{n-2i}). \end{aligned} \quad (3)$$

where $\alpha_i = a_{i,n-i}^u = a_{n-i,i}^d$. It is clear that for any given n and i , the R.H.S. contains a factor $(m_d - m_u)$. Therefore χ_{11}^{BI} (Eq.1) is proportional to m_2 if the higher order terms are sub-dominant. This is what we observed for the range of m_2 considered here.

For the higher order correlators one can similarly write,

$$\chi_{13}^{BI} = \frac{1}{24} (\chi_4^u - \chi_4^d + 2\chi_{13}^{ud} - 2\chi_{31}^{ud}) \quad (4)$$

$$\chi_{31}^{BI} = \frac{1}{54} (\chi_4^u - \chi_4^d - 2\chi_{13}^{ud} + 2\chi_{31}^{ud}) \quad (5)$$

$$\chi_{22}^{BI} = \frac{1}{36} (\chi_4^u + \chi_4^d - 2\chi_{22}^{ud}) \quad (6)$$

For all these quantities the first two terms on R.H.S. were found to be dominant. Considering again the Taylor expansion in quark masses, χ_{13}^{BI} and χ_{31}^{BI} were found to be proportional to m_2 . No such proportionality was found for χ_{22}^{BI} .

Finally let us discuss χ_{11}^{BI} for non-zero μ_B . In Fig.3 we present the results for three different temperatures - one above, one below and one close to the crossover temperature T_c . For large T , μ_B effects are small and χ_{11}^{BI} while remaining positive slowly approaches zero with increasing μ_B . Close to T_c χ_{11}^{BI} changes fast with increase of μ_B and in fact changes sign and then slowly approaches zero. This effect becomes much more prominent at lower temperatures and at the first order phase transition boundary changes sign almost discontinuously. The negativity of the correlator indicates that the isospin number decreases while increasing the baryon chemical potential. This is not expected for an ideal gas of quarks even with non-zero masses. Given $m_2 > 0$, we have $m_u < m_d$ and therefore it is expected that $\chi_{11}^{BI} > 0$. It is thus apparent that interactions play a major role in the high density matter.

An amazing fact remains that the scaling of the correlators with m_2 survives for all conditions of T and μ_B .

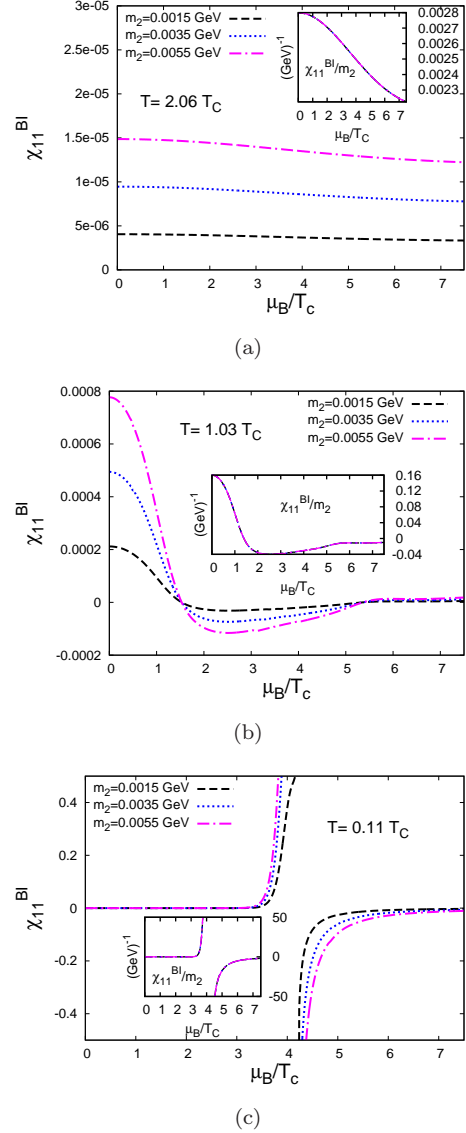


FIG. 3. Second order off-diagonal susceptibilities along chemical potential for three different temperatures.

This is shown in the insets of Fig.3. Now expanding χ_{11}^{BI} in a Taylor series in μ_B about $\mu_B = 0$ we have,

$$\chi_{11}^{BI}(\mu_B) = \chi_{11}^{BI}(0) + \frac{\mu_B^2}{2!} \chi_{31}^{BI}(0) + \frac{\mu_B^4}{4!} \chi_{51}^{BI}(0) + \dots \quad (7)$$

In the above series odd order terms vanish identically as χ^{BI} is CP even. Since $\chi_{11}^{BI}(\mu_B)$ on the L.H.S scales with m_2 , the same can be expected to hold true individually for all the coefficients on the R.H.S up to any arbitrary order. Incidentally the first two Taylor coefficients have already been shown to respect this scaling in Fig.1 and Fig.2(b) respectively.

Connection with Heavy Ion Collisions: Correlation between conserved charges, is an experimentally measurable quantity obtained from event-by-event analysis in

heavy-ion collisions. The sign change of χ_{11}^{BI} at high μ_B may be an important signature of phase transition in this region. This will of course need fluctuations to freeze out close to the critical value of μ_B .

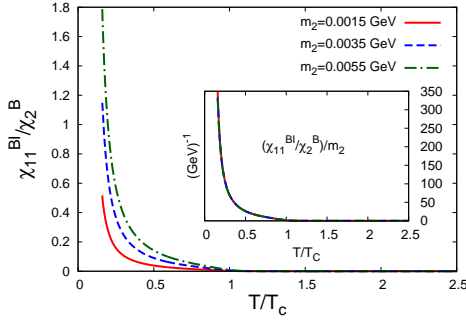


FIG. 4. Ratio of B–I correlation to baryon number fluctuation at $\mu_B = 0$.

To compare with experiments it is often useful to consider ratios like $R_2 = \chi_{11}^{BI}/\chi_2^B$. The temperature variation of R_2 is shown in Fig.4. It decreases monotonically to zero above T_c . Similar behavior can also be found with variation in μ_B and the ratio goes to zero outside the phase boundary. A systematic study of this ratio can thus indicate how close one could approach the phase boundary in heavy-ion collisions.

Also using the m_2 scaling one can estimate the mass asymmetry of constituent fermions in a physical system as, $m_2^{\text{expt}} = \frac{R_2^{\text{expt}}(T, \mu_B)}{R_2^{\text{PNJL}}(T, \mu_B)} \times m_2^{\text{PNJL}}$. To the best of our knowledge this is the first theoretical formulation that indicates that quark mass asymmetry in thermodynamic equilibrium can be directly measured from heavy-ion experiments. There still remains the question whether the isospin asymmetry brought in through QED effects may disturb the scaling.

Another important point to observe is that for fractional baryon number of the constituents $\chi_{13}^{BI} > \chi_{31}^{BI}$, and vice versa for integral baryon number i.e. for protons and neutrons. From Fig.2 we see that the former inequality persist well below T_c . This may well be an artifact of PNJL model. Thus it would be important to see corresponding results from Lattice QCD. Enhanced statistics of present and future experiments may make it possible to measure this extremely sensitive probe. The direction of the above inequality would be important in deciding if partonic matter may have been produced in the experiments.

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